Naval Research Laboratory

Washington, DC 20375-5320

NRL/FR/7210--06-10,127

The General Image Quality Equation and the Structure of the Modulation Transfer Function

Robert B. Hindsley
Lee J. Rickard

Radio/IR/Optical Sensors Branch
Remote Sensing Division

March 30, 2006

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*
30-03-2006

2. REPORT TYPE
Formal Report

3. DATES COVERED *(From - To)*
01-01-2004 to 06-01-2005

4. TITLE AND SUBTITLE
The General Image Quality Equation and the Structure of the Modulation Transfer Function

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER
0601153N

6. AUTHOR(S)
Robert B. Hindsley and Lee J. Rickard

5d. PROJECT NUMBER

5e. TASK NUMBER
SP034-06-41

5f. WORK UNIT NUMBER
72-7187

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Naval Research Laboratory
4555 Overlook Avenue, SW
Washington, DC 20375-5320

Office of Naval Research
One Liberty Center
875 North Randolph Street
Arlington, VA 22203-1995

8. PERFORMING ORGANIZATION REPORT NUMBER
NRL/FR/7210--06-10,127

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Naval Research Laboratory
4555 Overlook Avenue, SW
Washington, DC 20375-5320

Office of Naval Research
One Liberty Center
875 North Randolph Street
Arlington, VA 22203-1995

10. SPONSOR / MONITOR'S ACRONYM(S)
NRL and ONR

11. SPONSOR / MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES

14. ABSTRACT
Sparse aperture systems, despite their promise of lower weight or larger size, produce images that are noisier and more blurred when compared to images produced by a full aperture. Previous work by Hindsley and Mozurkewich (2001) showed that analysis of the Modulation Transfer Function (MTF) demonstrated the proportionality of signal-to-noise in a sparse aperture to the fill factor of the aperture. Analysis of the MTF also could enumerate the noise amplification characteristics of particular sparse apertures. However, such image quality metrics as the General Image Quality Equation (GIQE) also include edge effects, basically due to ringing and reduction in the edge sharpness. Here we report on our analysis of the MTF in order to quantify the relationship between the other terms in the GIQE and the structure of the MTF for high signal-to-noise ratio (SNR) imaging. We find that, for a fixed amount of optical surface, the image quality will improve with decreasing fill fraction due to an increase in resolution. Apodization of the Wiener Filter used to restore the image, as advocated by Hindsley and Mozurkewich, does not result in an improved image quality; use of the traditional unapodized Wiener Filter does improve the image quality. While the GIQE does not appear very sensitive to input SNR so long as SNR is high, the input SNR does limit the ability to successfully reconstruct the image and is the ultimate limiting constraint on reducing the fill fraction. The efficacies of different strategies for "tweaking" an optical system to improve the GIQE are presented. The onset of the failure to satisfactorily reconstruct the edges in an image depends on the particular type of array and MTF, as well as the SNR.

15. SUBJECT TERMS
image reconstruction   sparse aperture systems   Wiener filter

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
					Robert B. Hindsley
Unclassified	Unclassified	Unclassified	UL	15	19b. TELEPHONE NUMBER *(include area code)* (202) 404-4002

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. Z39.18

CONTENTS

THE GENERAL IMAGE QUALITY EQUATION AND THE STRUCTURE OF THE MODULATION TRANSFER FUNCTION

1. INTRODUCTION

Sparse aperture systems are lighter than a full aperture of the same aperture size; equivalently, a sparse aperture system will be larger in aperture, thereby producing greater theoretical resolution, than a full aperture system having the same weight. However, the images produced by sparse aperture systems are noisier and are less sharp compared to images produced by a full aperture. In Fienup [1] it was postulated that the exposure time for a sparse array, in order to produce a particular Fourier space signal-to-noise ratio (SNR), was proportional to f^{-3}, with f being the fill fraction. Lucke [2] also has explored the theoretical background of this. To better understand the ramifications of the Fienup Theorem, Hindsley and Mozurkewich [3] performed simulations in an attempt to understand this relationship and the conditions under which it held. Hindsley and Mozurkewich found that the Fienup Theorem held if the aperture configurations were scaled properly and if the image reconstruction were done with a Wiener filter but apodized in such as way as to obtain the same edge response.

However, the quality of an image does not depend only on the SNR of the image. The purpose of this paper is to understand how the image quality varies with the fill fraction of a sparse aperture. The General Image Quality Equation (GIQE; we use the version of Leachtenauer et al. [4]) was formulated to relate calculable system parameters (e.g., SNR and RER) to the practical experience of image analysts (i.e., the NIIRS scale), in an effort to determine not only what factors affected the image quality, but how strong the effects were. It enables the convenient prediction of system design impacts and of system performance under given operational conditions – at least within the domain of its assumptions. Our analysis is carried out within the framework of the GIQE; the appropriateness of using it for these purposes will be discussed below.

2. THE GIQE AND VARIATION WITH FILL FRACTION

The GIQE of Leachtenauer et al. [4] is

$$\mathbf{NIIRS_G} = 10.251 - a*\log10\ \mathbf{GSD} + b*\log10\ \mathbf{RER} - (0.656 * \mathbf{H}) \tag{1}$$

$$-(0.344 * \mathbf{G/\ SNR_{in}}),$$

$$a = 3.32 \text{ and } b = 1.559, \text{ for RER} \geq 0.9$$

$$a = 3.16 \text{ and } b = 2.817, \text{ for RER} < 0.9$$

with the following definitions:

$NIIRS_G$: image quality on the National Image Interpretability Rating Scales (NIIRS) numerical scale; defined from 0 to 9, with larger values indicating better images; we will refer to this as the GIQE value henceforth.

Manuscript approved September 7, 2005.

GSD: ground sampled distance (note that this is determined by the pixel size and can change independently of the resolution);
RER: relative edge response; related to the slope, as measured in the image, of an infinitely sharp edge in the object;
H: edge overshoot; a measure of ringing, as measured in the image, of an infinitely sharp edge in the object;
G: noise gain during reconstruction;
SNR_{in}: mean INPUT SNR.

It is important to note that this GIQE was defined for imaging systems in which Q = 1; Q, the diameter of the point spread function divided by the pixel width, is defined by

$$Q = (\lambda * f/D)/p = (\lambda * f) / (D * p) = 1, \tag{2}$$

with λ as the wavelength, f as the focal length, D as the enclosed aperture, and p as the pixel size. Q = 1 means that the image is undersampled, since Q = 2 for Nyquist sampling.

Fiete et al. [5] found that the GIQE did not describe well the reduction in image quality as fill fraction was reduced, although it is worth noting that they evaluated the GIQE using Q = 2 imagery. We have attempted to ameliorate these problems with the GIQE by restricting our analysis to a high SNR regime such that:

$$G/SNR << 1. \tag{3}$$

This means that the last term in the GIQE is small, and varies by a negligible amount. Requiring large values of SNR can be assumed to mean being photon noise limited. In practice, since the exposure time needed to obtain a particular SNR is proportional to f^{-3}, this might require impractically long exposure times for low fill fractions.

Even requiring that the SNR be high, there are at least three trade studies of imaging systems that could be done to see how image quality varies, depending on the nature and degree of the practical issues in implementation, and they could be expected to yield different answers. One study is to consider the case of having a fixed amount of optical surface, but variable aperture and fill fraction. A second study will consider the case in which the aperture size is fixed and the amount of optical surface and fill fraction is allowed to vary. A third study will examine the more general case of what differential change would produce the largest increase in image quality. In this report, we examine the first study in some detail in Section 3, developing techniques that will be used to draw conclusions for the second study (Section 5) and the third study (Section 6).

3. FIXED AREA OF OPTICAL SURFACES

First we consider the case is which the area of the optical surfaces is held constant, but the enclosed aperture of the system is allowed to increase. In this case, since the optical surface is constant, the fill fraction can be decreased only by increasing the aperture, and the fill fraction is inversely proportional to the square of the aperture. An increased aperture means that the resolution is improved (that is, the point spread function is narrower). Keeping Q = 1 requires decreasing the pixel size, or increasing the focal length (or an increase in wavelength, which may be a less desirable option). In any case, as long as Q = 1 and the aperture increases, the GSD will become smaller. For simplicity, assume the change is in the pixel size, so that the pixel size is inversely proportional to the aperture. Thus, pixel size is proportional to the square root of the fill fraction, and likewise for the GSD. The upshot is that the variation in GSD is related to fill fraction by:

$$\mathrm{GSD} \propto f^{1/2}. \tag{4}$$

However, the variation in RER and H with fill fraction cannot be easily predicted or characterized. These must be investigated with actual experiments or simulations. While complicated standard patterns are often used as objects in such simulations, we chose to use the simple image shown in Fig. 1. In this image, all the pixels are zero except for the "central half" of the range in each coordinate, where the pixels all have values of 1. That is, for a 1024 by 1024 pixel frame, the central block of pixels with *both* $256 < x < 768$ *and* $256 < y < 768$ are 1, and all other pixels are zero. This simple image ensures that there are no overlapping effects between edges. The RER and H were evaluated at the middle of each face of this central block.

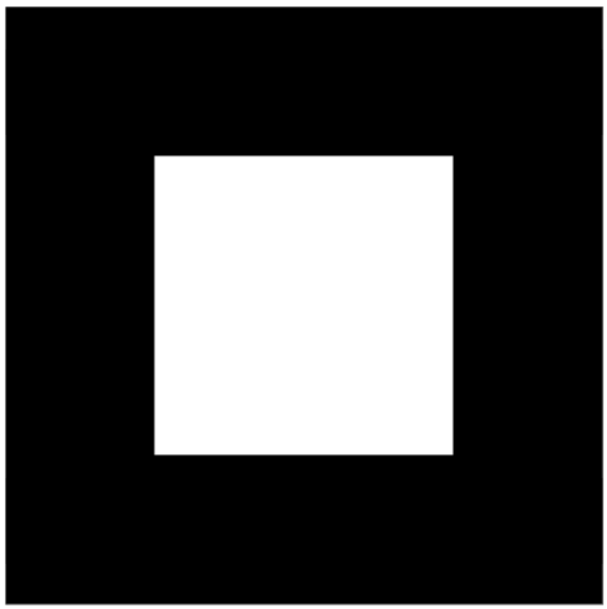


Fig. 1 — Image used for evaluation of edge parameters RER and H. Pixels in the outer dark region are equal to zero, while pixels in the light inner square are equal to 1. The inner square is ¼ of the entire frame, with each side equal to ½ of the whole frame, and the square is centered at the center of the frame.

An annulus is the simplest sparse aperture and a convenient beginning. Figure 2 shows how the GIQE value changes with fill fraction for images with SNR = 100, as well as the change in the GSD and edge (RER+H) component of the GIQE. On these plots, the ordinate value GIQE = 0 would be realized if the image had the GSD of a filled aperture but an infinitely sharp edge, with RER = 1 and H = 0. There is, in fact, no such image possible, as an infinitely sharp edge would need an infinitely large aperture. However, this does form a convenient reference, which will be referred to as the reference image.

For a filled aperture, the edge is not infinitely sharp, so the terms in the GIQE due to RER and H are decreased by almost a full unit from the reference image. As the fill fraction is decreased, the increase in aperture improves the resolution and decreases the GSD, which increases the GSD term (plotted as triangles) in the GIQE. If no image reconstruction is done, Fig. 2(a) shows that the decrease in fill fraction causes the edge to get progressively less sharp, causing the RER and H terms in the GIQE (plotted as diamonds) to decrease. The edge terms dominate the GSD term, and the overall GIQE (plotted as a solid line) continuously decreases as the fill fraction decreases.

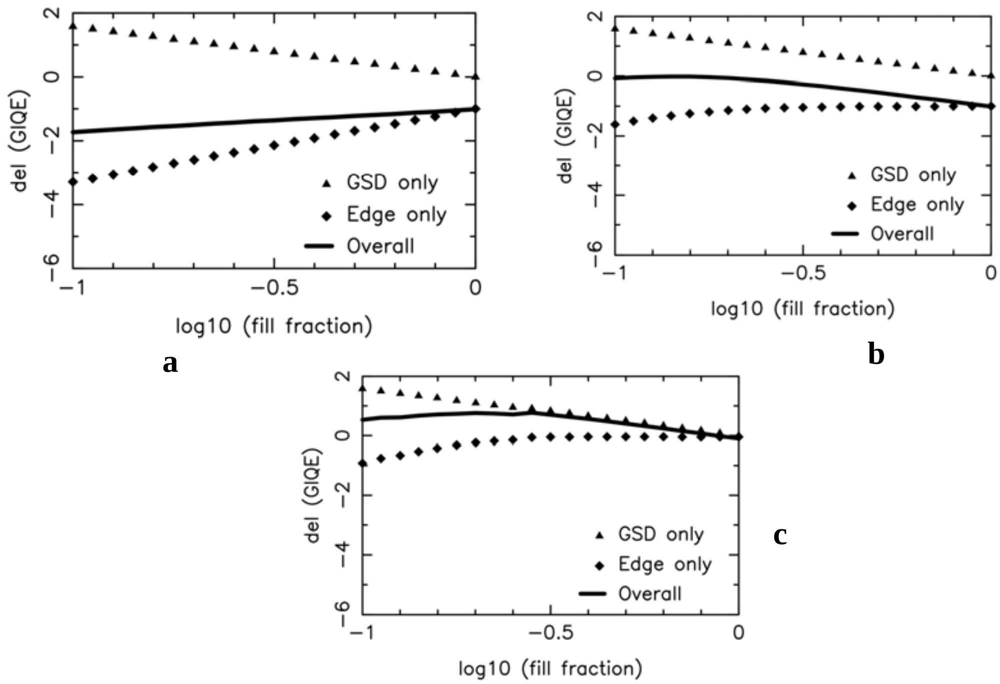


Fig. 2 — Plot of change in GIQE as a function of fill fraction for annuli: a) for unreconstructed images, b) for apodized reconstructions, and c) for unapodized reconstructions. The triangles are the change in GIQE due to GSD only. The diamonds are the change due to edge response only. The solid line is the total change in GIQE.

Hindsley and Mozurkewich [3] found that reconstruction with an apodized Wiener filter (apodized using a full-aperture MTF) gave improved suppression of noise relative to the signal in reconstruction, compared to a traditional Wiener filter. The apodized Wiener filter is given by:

$$WF(u,v) = A(u,v)\ MTF(u,v)\ /\ [\ (MTF\ (u,v)\ )^2 + C^2\ ], \tag{5}$$

where MTF(u,v) is the modulation transfer function, C is a constant usually taken to be 1/(mean SNR of the image), and A is an apodizing function. The traditional Wiener filter (no apodization) simply has A(u,v) = 1 for all (u,v). The traditional Wiener filter attempts to increase the response to unity at all frequencies, which can lead to unfortunately large amplification of the noise. Use of an apodizing function implies that the response at some frequencies will be reduced, compared to the response of a traditional Wiener filter. Apodization can be used to remove regions of the MTF that provide excess noise. As in Hindsley and Mozurkewich [3], the apodizing function A(u,v) used here will be a full-aperture MTF.

Certainly the apodized Wiener filter reconstructions in Fig. 2(b) give a better result than with no reconstruction (Fig. 2(a)). The GIQE for the filled aperture reconstructed apodized image is the same as for the unreconstructed image from the filled aperture; indeed, the images themselves are nearly identical. However, as the fill fraction decreases to about 0.3, the GIQE increases until the reconstructed image has a GIQE almost two units higher than the unreconstructed image at the same fill fraction. This is due to the edge response terms maintaining their filled aperture values for fill fractions greater than about 0.3, while the decrease in GSD at those fill fractions improves the overall GIQE. If the fill fraction is decreased below about 0.3, the reconstructed edge is no longer as sharp as it was for the filled aperture, and the edge terms in the GIQE decrease. This decrease is noticeable when RER is still as high as 0.9;

RER = 0.9 is also significant because two of the coefficients in the GIQE given in Eq. (1) change. This produces a discontinuous decrease in the GIQE of about 0.1. While a change of 0.1 is an insignificant decrease in the GIQE, and it is somewhat artificial due to the change in coefficients, this value of RER does make for a convenient benchmark.

As the fill fraction is reduced further in Fig. 2(b), and the RER and H terms in the GIQE decrease, this is roughly balanced by the increase in GIQE due to the improvement caused by reduction in GSD. The GIQE is almost as good as the reference image, which has infinitely sharp edges and the GSD of a filled aperture. While the image is not the same as the reference image, it seemingly has as much information.

However, best of all are the traditional (that is, apodized) Wiener filter reconstructions as shown in Fig. 2(c). First, for the filled aperture, the overall level is higher by a full unit than for the other cases with filled aperture. This is because the traditional Wiener filter boosts the system response beyond that of the filled aperture, and attempts to reproduce the frequency response needed to provide infinitely sharp edges. As the fill is decreased, the edge sharpness is essentially maintained, and the increase in aperture and decrease in GSD leads to an improvement in GIQE, so that the image has more information than the reference image! Again, with the fill below about 0.3, the edges cannot be reconstructed so sharply, and the GIQE begins to decrease slightly as fill fraction decreases.

The treatment of Hindsley and Mozurkewich can be misunderstood: while an apodized Wiener filter does a better job of avoiding the overamplification of noise, it does NOT give better edge response, and therefore doesn't give better images.

To show that the results are the same for other configurations, the analysis is repeated with a Golay-like array of nine apertures developed by Coleman [6] and referred to in his notations as a Golay-9 (2,15,16) array. Table 1 gives general positions of the individual apertures, which have to be scaled to give configurations having particular fill fractions. The individual apertures can be no larger than 1 unit, at which point several of them touch; in this situation, the configuration has the maximum possible fill fraction of 0.36. A diagram of a Golay-9 array, but with fill fraction of 0.25, is shown in Fig. 3. For the array of Fig. 3, the fill fraction has been decreased by increasing the aperture separation by a factor of 1.25 while maintaining the diameter of the individual apertures.

The results of the image quality analysis for these Golay-9 arrays are given in Fig. 4. As with the annuli, the GIQE value is seen to increase so long as the images can be reconstructed with edges having RER > 0.9.

4. WHY SNR IS IMPORTANT

What is the limit to the increase of GIQE value with decreasing fill fraction? As the fill fraction is reduced for any sparse aperture configuration, eventually regions appear in the MTF with negligible response, information is irretrievably lost, and the edges become progressively less sharp as the fill fraction is decreased further. This can be seen in Fig. 5. In Fig. 5(a), the GIQE values increase with decreasing fill fraction so long as the edge component of GIQE is unchanged. But as some fraction of the MTF values become roughly equal to C (which equals 1/SNR) in Eq. (5), the edges produced by reconstruction become less sharp, and the GIQE value begins its decrease with decreasing fill fraction. In

Table 1 — Positions of the Centers of the Nine Apertures for a Golay-9 (2,15,16) Array

Aperture	X	Y
1	-0.5	1.443
2	2.0	0.577
3	1.5	1.443
4	-1.0	-1.155
5	-1.5	1.443
6	-2.0	0.577
7	1.5	-0.289
8	-0.5	-2.021
9	0.5	-2.021

Fig. 5(b), the analysis is repeated with the input SNR decreased from 100 to 10; that is, the term C in Eq. (5) is *increased* from 1/100 to 1/10. As expected, with a lower SNR, the GIQE value begins to decrease at a higher fill fraction; regions with value approximately equal to 1/SNR appear in the MTF at a higher fill fraction for lower values of SNR.

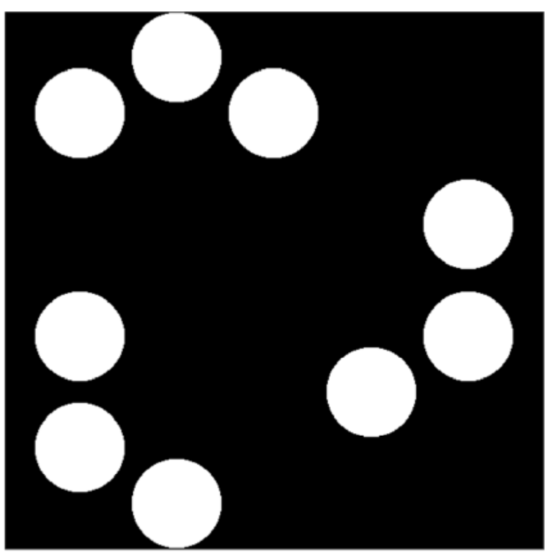
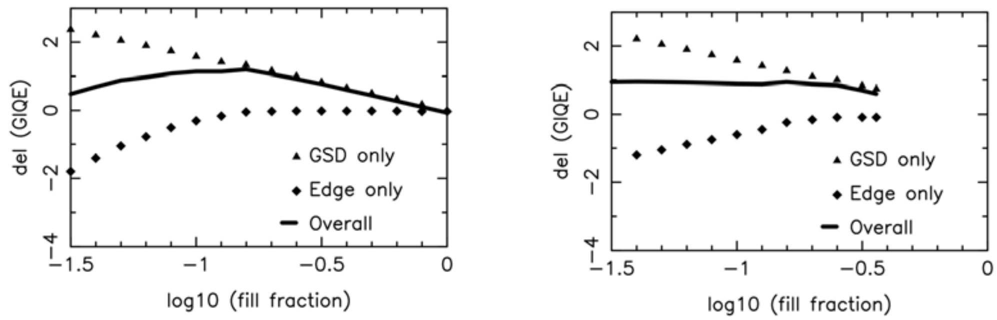


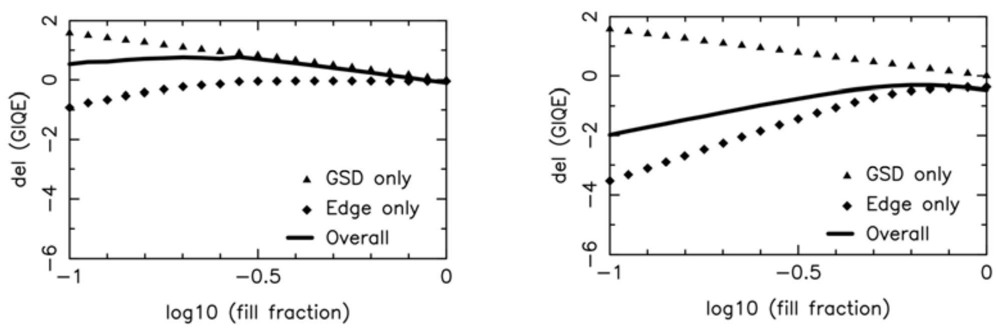
Fig. 3 — Golay-9 (2,15,16) array of 9 apertures, with fill fraction = 0.25. The light disks are the nine individual circular apertures with unit diameter. The positions of the aperture centers given in Table 1 have been multiplied by 1.25; this scaling yields the field fraction of 0.25.

As noted before, the explicit role of the SNR in the GIQE equation seems very minimal so long as SNR is sufficiently greater than the noise gain G. However, this condition that SNR>>G is in fact the crux of the matter, as it determines how low the fill fraction can go before Wiener filter reconstruction cannot yield images with RER > 0.9 and the image quality is seen to degrade. This roundabout dependence on SNR only manifests itself when reconstructions are done, and does not reflect a problem with the GIQE itself (see Fig. 2(a), in which the GIQE consistently decreases as fill fraction decreases). However, the improvement in image quality seen when comparing Figs. 2(a) and 2(c) shows why image reconstructions are always performed.



a **b**

Fig. 4 — Plot of change in GIQE as a function of fill fraction for annuli (a, illustrating the same data as Fig. 2(c)), and for a Golay 9-(2,15,16) array (b). The triangles are the changes due to GSD only. The diamonds are the change in GIQE due to edge response only. The solid line is the total change in GIQE.



a **b**

Fig. 5 — Plots of change in GIQE as a function of fill fraction for annuli (a, identical to Fig. 2(c), is for images with input SNR = 100, and b is for images with input SNR= 10). The triangles are the change in GIQE due to GSD response only. The diamonds are the changes due to edge response only. The solid line is the total change in GIQE.

5. FIXED APERTURE SIZE

Section 3 considers the case of having a fixed amount of optical surface. A different case would be to have the longest baseline fixed while fill fraction is changed by varying the amount of optical surface. For example, an annulus would have its outer diameter fixed while the inner hole varied in size. A Golay array would be more complicated: shrinking the individual elements would slightly decrease the longest

baseline, so the element spacing would have to be increased slightly to maintain the longest baseline while decreasing the fill fraction. A limit on the maximum longest baseline might be due to size limitations on the overall imaging package due to launch vehicle limitations, for example. The previous case demonstrates that the longest baseline should generally be as large as possible to provide the highest resolution and smallest GSD. The resolution is determined by the longest baselines, so that the GSD term in the GIQE would not change with the fill fraction.

What is to be gained by maximizing the fill fraction in such a case? While we have ignored the SNR term in the GIQE, on the assumption that the exposure time can be increased to achieve any reasonable desired SNR, it is almost always an advantage to minimize the exposure time. Minimizing the exposure time is equivalent to adding optical surface area and, therefore, maximizing the fill fraction. In general, other issues might justify a small loss in image quality. One obvious example is weight — a reduction in optical surface generally means a corresponding reduction in weight, both from having less “glass” and from having a lighter support system. On the other hand, decreasing the fill fraction may mean that the support system has to be more complex, and alignment will almost certainly be a more difficult problem.

In short, the GIQE would predict that reductions in the fill fraction, over some range, would not produce much deterioration in image quality, so long as the image restoration results in edges with RER > 0.9. Only the desire to minimize exposure time would push for maximizing the fill fraction.

6. MAKING A DIFFERENTIAL CHANGE

The third trade study to be done is to consider the case of an optical system that can be “tweaked” slightly. What small change in parameters such as fill fraction, enclosed aperture, or amount of optical surface, might give the greatest increase in GIQE?

Examination of Fig. 2(c) suggests that, for the fill fraction regime in which the edge response can be restored by reconstruction so that RER> 0.9, decreasing the GSD would give the largest increase in GIQE. This would mean increasing the aperture, but need not include an increase in the amount of optical surface to maintain the fill fraction. In Fig. 2(c), for an annulus with fill fractions above 0.3 (-0.5 in the log), decreasing the fill fraction slightly would still give the same edge response term in the GIQE. This does assume that the SNR is maintained sufficiently high (by increasing the exposure time if necessary) so that the image reconstruction does yield RER > 0.9. Therefore, assuming that the SNR remains sufficiently high, the conclusion is that spreading the optical surface slightly would be the most effective tweak that could be done to improve image quality.

Is this true at lower fill fractions? At fill fraction = 0.2 (-0.7 in the log), increasing the longest baseline without adding optical material to maintain the fill fraction would yield no improvement in GIQE. There is a loss in GIQE due to loss of edge response that counteracts the gain due to decrease in GSD. In this fill fraction regime, a better strategy would be to add optical surface within the fixed enclosed aperture in order to increase the fill fraction. Keeping the enclosed aperture unchanged would mean that the GSD is unchanged, but an increase in the fill fraction would improve the edge response. Adding the optical surface would improve the SNR slightly, and the improvement in the SNR will also improve the ability to reconstruct the image. This could be a significant effect if the optical system happens to be very near the fill fraction at which image reconstruction starts to yield edges with RER<0.9. Thus, in the case of a fill fraction so low that image reconstruction is a problem, the most effective tweak is to add optical surface to improve the fill fraction while maintaining the GSD.

7. GIQE AS A FUNCTION OF MTF AND MEAN SNR

Is there a general relationship between the MTF and the mean SNR that can be used to predict what combination of parameters gives the best possible image quality? Examination of Eq. (5) will show that in those regions of the MTF where the MTF value is much larger than 1/SNR, the denominator is roughly $(MTF)^2$, and the Wiener filter is close to 1/MTF. It is certainly the case that the RER value drops to 0.9 when parts of the MTF become so reduced that the "noise term" in the denominator of the Wiener filter causes the filter to be significantly different from 1/MTF for those frequencies. One might guess that this occurs when some fraction of the MTF is equal to or less than 1/SNR.

This guess can be tested. For a particular MTF, as the mean SNR is decreased, the RER term in the GIQE of the reconstructed image eventually will decrease also. For annuli with different fill fractions, the mean SNR was adjusted until the RER was equal to 0.9 after image reconstruction. For each of these combinations of fill fraction and SNR, the relative frequencies of different values of the pixels in the MTF were compiled ("pixels," by analogy with an image, refers to the discrete elements representing particular frequencies in the MTF). The values of these MTF pixels are best considered when divided by the SNR, as the denominator of the Wiener filter implicitly compares the pixel value to 1/SNR.

An example of distribution of frequencies of the pixel values in the MTF of an annulus is shown in Fig. 6. For any annulus, the pixels in the MTF tend to have values within a fairly narrow band of about 2/SNR width. There is also a spike in frequency at the lower limit of this band; in this particular case, the spike in the pixel value frequency distribution is at about 2.4/SNR.

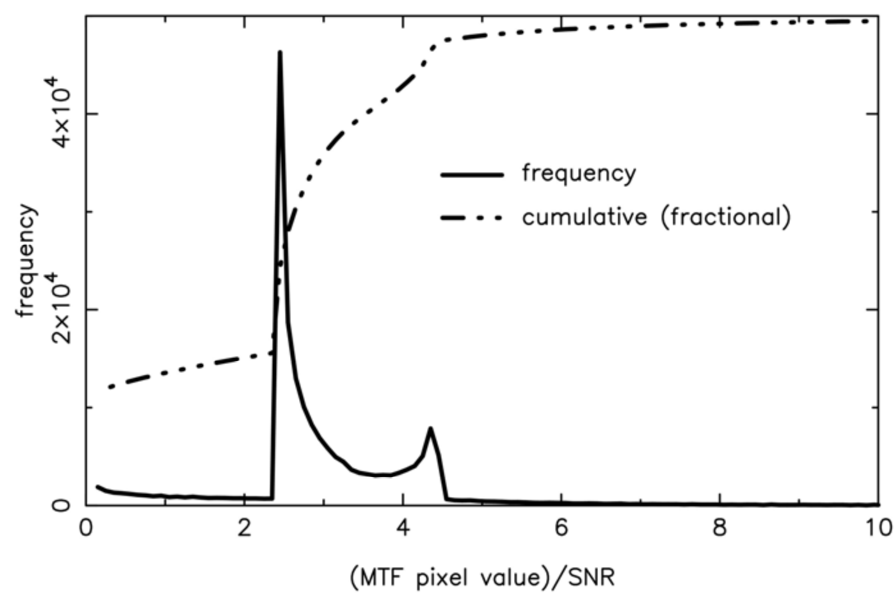


Fig. 6 — Plot of pixel frequency in the MTF of an annulus with a fill fraction equal to 0.316 and SNR of 70.8, which gives RER = 0.9 in the reconstructed image. The abscissa is the MTF pixel value divided by SNR. The solid line is the frequency of occurrence of each pixel value. The broken line is the cumulative frequency of pixels below that value, expressed as a fraction of the total number of pixels. The total number of pixels in the MTF is 262 144.

If this ratio 2.4/SNR held for all array types and fill fractions, it would be a very useful ratio. However, it does not hold even for annuli with other fill fractions. Table 2 presents the results of similar

analysis for annuli with a range of fill fractions. Not surprisingly, the SNR for which RER equals 0.9 is found to increase as the fill fraction decreases. But the ratio of the MTF pixel value to SNR at the spike in frequencies decreases as the fill decreases. The decrease in this ratio is rather abrupt at fill fractions over 0.5 and seems to be asymptotically approaching a value that is roughly 2.

Table 2 — For Annuli with the Given Fill Fractions, the SNR that Yields RER = 0.9 After Image Reconstruction, and the Lower Limit of the Band of MTF Pixel Values (Expressed as Multiples of 1/SNR)

Fill Fraction	SNR	lower limit of pixel band
0.794	31.6	7.25
0.501	39.8	3.1
0.316	70.8	2.45
0.178	158	2.05
0.100	398	2.15
0.0568	891	1.95

This behavior also appears when considering Golay 9 arrays. In this case, the range of the fill fraction is limited from both extremes, as it cannot be greater than 0.36 (or the individual elements touch), and at 0.10 there are zeros in the MTF that make it impossible to achieve RER = 0.9 (the edge is not sharp enough even after image reconstruction). Figure 7 shows a plot of the frequencies of pixel values in the MTF. Like the plot of the frequencies of pixel value in the MTF of annular arrays, there is a large spike in the frequency of pixel values (here at MTF pixel value/SNR = 5.65), but with the Golay 9 there is no sharp upper limit delineating a band. Table 3 shows how the SNR increases, and the MTF pixel value at which the spike occurs decreases, as the fill fraction decreases. Again, the spike seems to asymptotically approach a particular value — about 5.5 in this case.

It seems that any particular configuration exhibits this behavior — the value of the spike in frequency of MTF pixel/SNR, for that SNR that gives RER = 0.9 in the reconstructed image, approaches some asymptotic value as the fill decreases. What this value is must depend on the details of the MTF structure in a rather complex way, for this reflects the failure of the various spatial frequencies to be properly included in reconstruction of the edges.

8. CONCLUSION

In high SNR imaging, defined as G/SNR<<1 in the GIQE, it is found that as the fill fraction is decreased for a fixed amount of optical surface, the image quality for a Wiener filter-reconstructed image will improve due to the decrease in GSD. While previous work showed that an apodized Wiener filter gave better noise response in image reconstruction, the traditional Wiener filter gives better overall image quality because of better edge response. While the GIQE does not appear very sensitive to input SNR on the surface, the input SNR limits the ability to obtain sharp edges in the image reconstruction, and sets the ultimate lower limit on the fill fraction. The performance of an optical system may best be improved

slightly by increasing the aperture if the fill fraction is large, or by adding optical material to increase the fill fraction if the fill fraction is small. The distribution of (MTF pixel value)/SNR shows a spike that appears at a lower value of (MTF pixel value)/SNR as the fill fraction is decreased. The particular value at which this spike occurs also depends on the array type.

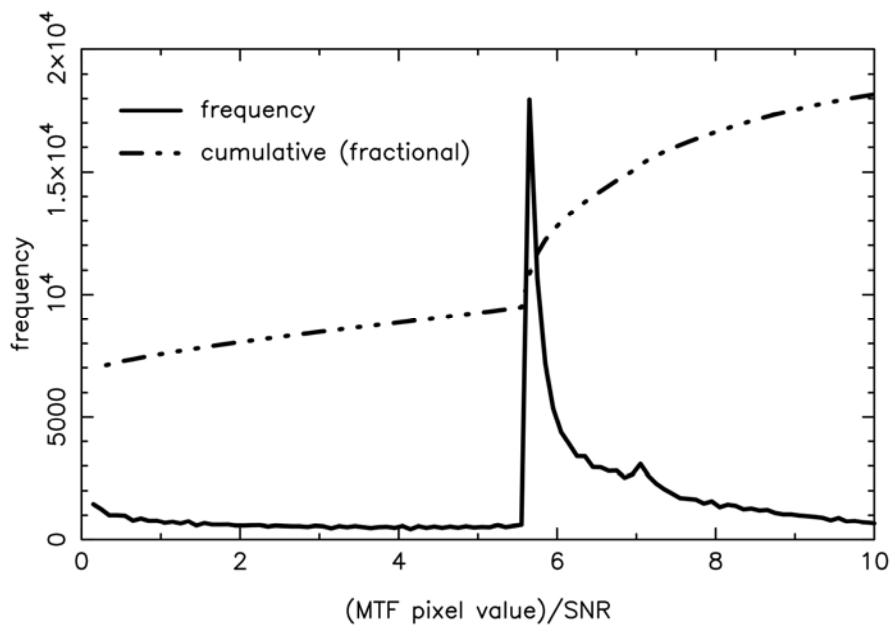


Fig. 7 — Plot of pixel frequency in the MTF of a Golay9-(2,15,16) with a fill fraction equal to 0.25 and an SNR of 200, which gives RER = 0.9 in the reconstructed image. The abscissa is the MTF pixel value divided by SNR. The solid line is the frequency of occurrence of each pixel value. The broken line is the cumulative frequency of pixels below that value, expressed as a fraction of the total number of pixels. The total number of pixels in the MTF is 262 144.

Table 3 — For Golay 9-(2,15,16) Arrays with the Given Fill Fractions, the SNR that Yields RER = 0.9 After Image Reconstruction, and the Lower Limit of the Band of MTF Pixel Values (Expressed as Multiples of 1/SNR)

Fill Fraction	SNR	lower limit of pixel band
0.350	178	10.25
0.316	178	8.6
0.250	200	5.65
0.150	891	5.55
0.100	no SNR yields RER=0.9	-------

9. ACKNOWLEDGMENTS

The authors wish to thank Dr. David Mozurkewich of Seabrook Engineering for suggesting this project and for providing much useful software, and also Dr. Robert Lucke of NRL for many insightful comments.

REFERENCES

1. J.R. Fienup, “MTF and Integration Time vs. Fill Factor for Sparse Aperture Imaging Systems,” in *Imaging Technology and Telescopes*, J.W. Bilbro et al., eds., *Proc. SPIE* 4091A-06, San Diego, CA, 30 July 2000, pp. 43-47.

2. R.L. Lucke, “Fundamentals of Wide-Field Sparse-Aperture Imaging,” paper 5.0101, *Proceedings of the IEEE Aerospace Conference*, Big Sky, Montana, March 10-17, 2001.

3. R.B. Hindsley and D. Mozurkewich, “Signal to Noise in Sparse-Aperture Imaging,” paper 5.0103, *Proceedings of the IEEE Aerospace Conference*, Big Sky, Montana, March 10-17, 2001.

4. J.C. Leachtenauer, W. Malila, J. Irvine, L. Colburn, and N. Salvaggio, “General Image Quality Equation: GIQE,” *Appl. Opt.* **36**(32), 8322-28 (1997).

5. R.D. Fiete, T.A. Tantalo, J.R. Calus, and J.A. Mooney, “Image Quality of Sparse Aperture Designs for Remote Sensing,” *Opt. Eng.* **41**(8), 1957-1969 (2002).

6. John Coleman, private communication.